Engineering Notes

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Hot Surface—Cold Gas Boundary Layer with Heterogeneous Combustion

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Introduction

THE purpose of this investigation is to theoretically model the environmental conditions imposed on graphite test speciments by the "hot wall—cold gas" experimental test facilities. These situations exist, for example, when the specimen is resistively heated in the presence of air at moderate temperatures and low (subsonic) flow rates. In other cases, the mode of heating may be radiative from an external source, also employing slowly moving, low temperature air to provide oxygen and remove the ablation products. If, in these latter cases, neither the air nor the ablation products absorb significant portions of the radiative energy flux, then the model will interact with the gaseous environment in precisely the same manner as if it had been internally heated by electrical resistance. In these situations where the graphite specimen is operating at the temperatures necessary for oxidation and vaporization, the lower temperature test gas is being heated in a thermal boundary layer by the condensed phase.

To model the testing situations discussed, a coupled set of multicomponent, chemically reacting, boundary-layer equations is utilized. The chemical model for graphite oxidation utilized in this present study is discussed in detail in Ref. 1-3. The oxidation model includes the following six species: 0, 0_2, N, N_2, C0, and $C0_2$. The treatment of the system of differential equations and resulting solutions for the oxidation of graphite in hypersonic re-entry environments is given in Refs. 1-3. The outer boundary conditions applied in Refs. 1-3 were such that the gas temperature and enthalpy at the edge of the viscous layer were greater than the gas temperature and enthalpy at the gas-solid interface. In this present work, however, stagnation point solutions are obtained for the boundary conditions $T_w > T_e$ and $h_w > H_e$.

Numerical Results

Typical boundary-layer results obtained in this study are shown in Figs. 1-3. It was found, in agreement with earlier studies,[4-7] that for these boundary conditions the velocity of the gas inside the boundary layer (Fig. 1) can exceed the local freestream value. Note that in spite of the fact that $T_w > T_e$ and $h_w > H_e$, the temperature gradient is positive at the surface for these surface temperatures. The increase in temperature away from the surface is due to the homogeneous exothermic combustion reaction. It should be noted, however, that the net heating is directed away from the surface and into the gas phase. For this situation, the energy transfer away from the surface due to diffusion and convection dominates the portion of energy transfer due to conduction. The enthalpy undershoot for this particular wall temperature is caused by the high rate of energy transfer away from this region of the boundary layer. The composition profiles (Fig. 2) show that the species $C0_2$ plays a far more important role in the oxidation process for low edge temperatures than it does for the hypersonic re-entry boundary conditions. In the present case, the magnitude of $C0_2$ in the boundary layer is about three times larger. In addition, for these conditions, the species C0 vanishes quite close to the surface, and $C0_2$ is the

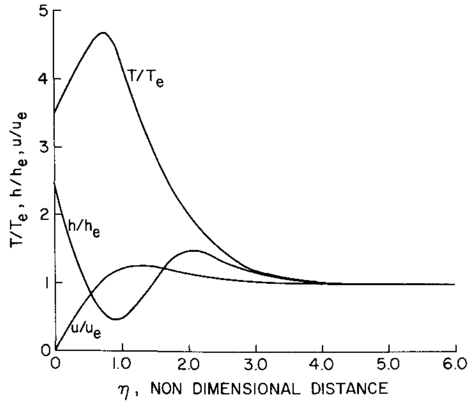

Fig. 1 Temperature, velocity, and enthalpy profiles. $P_e = 100$ atm, $H_e = 110$ btu/lb, $T_w = 3500°R$.

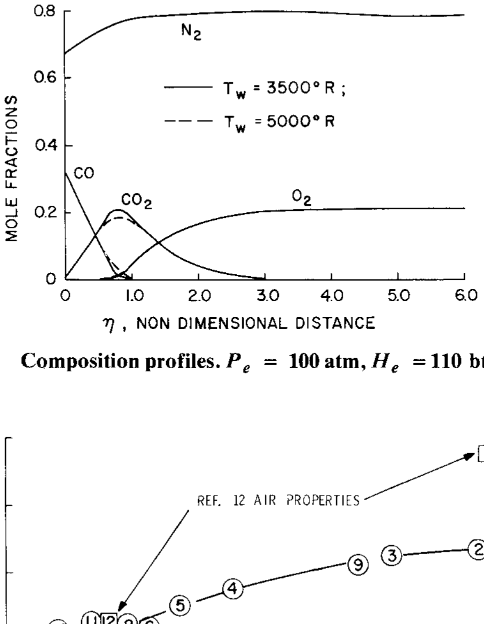

Fig. 2 Composition profiles. P_e = 100 atm, H_e = 110 btu/lb.

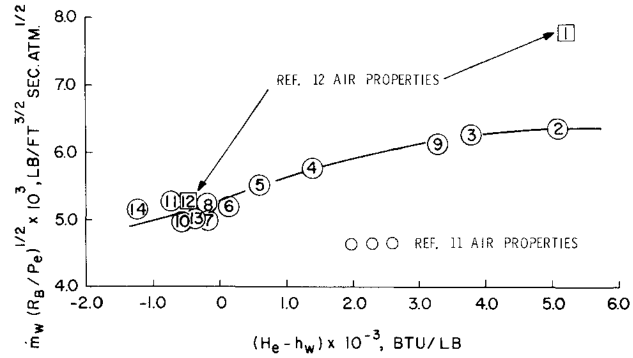

Fig. 3 Diffusion-controlled graphite oxidation, stagnation point mass transfer parameter. Note: Numbers indicate results obtained from boundary-layer solutions.

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predominate ablation species in the outer regions of the viscous layer.

A correlation of a number of boundary-layer solutions for a wide range of boundary conditions (i.e., $3500 < T_w < 7000°R$, $10.0 < P_e < 100$ atm, and $100.0 < H_e < 6000.0$ btu/lb) led to the results shown in Fig. 3. In this figure, the stagnation point mass transfer parameter $\dot{m}_w (R_B/P_e)^{1/2}$ is plotted as a function of the enthalpy difference $(H_e - h_w)$ across the boundary layer. These results indicate that when $H_e < h_w$ the diffusion controlled oxidation rate is reduced by as much as 20% from the rate which would be predicted for hypersonic re-entry conditions (i.e., where $H_e \gg h_w$). This reduction in oxidation rate for $H_e < h_w$ has been observed experimentally.[8-10] It should be noted, however, that theoretical calculations for graphite vaporization with "hot-wall—cold gas" boundary conditions yielded no significant deviation in the magnitude of the mass transfer rate from that which would be predicted for hypersonic re-entry.[2-3]

Discussion

The numbers on Fig. 3 indicate the computer run number for the boundary-layer solutions obtained during this study. The differences obtained in the mass transfer parameter due to differences in the transport properties utilized are clearly indicated by the results of Run #1 and #2. The properties for the air species used in Run #2 are based on the Scala-Baulknight model.[11] The higher oxidation rate calculated in Run #1 is due to the higher binary diffusion coefficients for the atomic species which are predicted by the Yun and Mason[12] atomic cross sections. Compare this, however, with the oxidation rates calculated from Runs #11 (Ref. 11 properties) and #12 (Ref. 12 properties). Now for these later boundary conditions (i.e., Runs #11 and 12), the air at the edge of the boundary layer is not dissociated as it was for Runs #1 and #2. When the edge temperature is low, atomic oxygen is not present to enter into the combustion reaction, and for this situation the properties of Refs. 11 and 12 for air lead to the same prediction for the oxidation rate.

The explanation for the reduction in the oxidation rate for the "hot wall—cold gas" boundary conditions is related to the magnitude of the species CO_2 which can be produced within the boundary layer away from the oxidizing surface. Recall that the expression for the oxidation $(\dot{m}_w)$ rate is defined[1,3] in terms of diffusion fluxes (j_i) and mass fractions (c_i)

$$\dot{m}_w = (j_{CO} + j_{CO_2})_w / (1 - C_{CO} - C_{CO_2})_w \tag{1}$$

and for the purposes of clarifying this point, let us consider the Fick's Law approximation for the diffusion flux vector.

$$j_{i_w} \cong -\left(\frac{\mu Le}{Pr} \nabla C_i\right)_w \tag{2}$$

where μ, Le and Pr are the viscosity and Lewis and Prandtt numbers of the gas mixture, and ∇C_i is the concentration gradient. Now it has been demonstrated[1,3] that for the equilibrium diffusion-controlled oxidation of graphite, it is the species CO_2 which diffuses back to the surface and provides the oxygen for the heterogeneous reaction $CO_2 + 2\,C(s) \rightarrow 2\,CO$. The carbon monoxide which is formed then diffuses away from the surface. Note that the composition profiles (Fig. 2) show that the gradient of the species CO_2 is positive, and the gradient of CO is negative at the surface. This means that the diffusion flux [Eq. (2)] of CO_2 is negative while the diffusion flux [Eq. (2)] of CO_2 is negative while the diffusion flux of CO at the surface is positive. Examination of Eq. (1) shows that increasing the diffusion flux of CO_2 tends to reduce the overall oxidation rate. One can conclude, therefore, that for these boundary conditions where the magnitude of the species CO_2 can be maximized in the boundary layer, its gradient and diffusion flux at the surface is also increased, leading to a reduction in the overall oxidation rate.

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Exponent Breaks in Composite Solid Propellants

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IT has been noted by Miller, Hartman, and Myers[1] that the Summerfield granular diffusion flame (GDF) rate-pressure relationship[2] implies

$$\lim_{d_{ox} \to \infty} r = 0 \tag{1}$$

where r is burning rate and d_{ox} is oxidizer particle size. Since this limit is physically unattractive, they modified the GDF relation to

$$r = ap^s + (b/p + cd_{ox}/p^{1/3})^{-1} \tag{2}$$

where p is pressure and a,b,c, and s are constants; s and a were identified with the binder. However, for composite

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